



## ON THE DESIGN OF LEN LYE'S FLAMING HARMONIC

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### Abstract

Len Lye (1901-1980) was an artist who wanted to build kinetic sculptures at a monumental size however he was constrained by the availability of resources and technology. This paper examines the structural design of a vertical cantilever beam which is a key element in the monumental version of Lye's 'Flaming Harmonic'. In 'Flaming Harmonic' the vertical cantilever vibrates at resonant bending and whirling frequencies due to a reciprocating harmonic ground movement which is provided by a mechanical drive. Lye produced a 1.3m tall model of a similar work called 'Harmonic'. Design rules developed in this paper show that 'Flaming Harmonic' can be produced at a height of 14m. Similarity between 'Harmonic' and 'Flaming Harmonic' ensures that while the artwork is produced posthumously, artistic integrity for the larger work will be preserved. The vibratory mode shapes and corresponding stresses are established for 'fixed' and 'pivot' clamp designs. It is shown that while a slightly different vibrating shape is achieved using each method the 'pivot' clamp design results in a lower bending stress and hence a longer life for the vertical cantilever.

**Keywords:** Collaborative design, Design practice, Optimisation, Simulation

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## 1 INTRODUCTION

New Zealand-born artist Len Lye (1901-1980) was internationally known for his work as an experimental filmmaker and kinetic sculptor. A common theme in Lye's work is the concept of 'composing motion'. He believed that if there was such a thing as composing music then there should be a means for composing motion (Horrocks, 2002). This idea was realised in many of Lye's artworks, produced from the late 1920s through to the early 1970s.

Lye developed concepts for some very large sculptures with complex kinetic behaviours, but he was constrained by technology, cost, and availability of materials. Unable to build his sculptures at full size, Lye instead prepared sketches, detailed notes, and smaller-scale models, which he intended to have made on a monumental scale when the technology and materials became available. Following Lye's death, the Len Lye Foundation was established to preserve and promote his legacy. Researchers at the University of Canterbury's Department of Mechanical Engineering in New Zealand have been working alongside the Len Lye Foundation to establish the implications for building some of Lye's proposed artworks. This collaboration between the fine arts and mechanical engineering has resulted in the posthumous completion of some of Lye's sculptures including 'Big Blade' (1998, 3.7m tall, Christchurch, NZ), 'Wind Wand' (2001, 48m tall, New Plymouth, NZ), and 'Water Whirler' (2005, 12m tall, Wellington, NZ). Interest in Lye's work continues to grow, and in July 2015 the Len Lye Centre was opened in New Plymouth as an archive and exhibition space for Lye's sculptures, films, paintings and other artworks.

This paper concerns the design and analysis of a new Len Lye sculpture called 'Flaming Harmonic'. The study will establish the feasibility of building a full-size 'Flaming Harmonic', at between 10-14m tall. 'Flaming Harmonic' is similar to another Len Lye sculpture called 'Water Whirler' (Figure 1), with the addition of flames. Like 'Water Whirler', 'Flaming Harmonic' consists of a hollow vertical cantilever rod which is excited at the base to form modes of vibration. As 'Flaming Harmonic' vibrates, water jets or gas flames are emitted from jets incorporated along the length of the cantilever.



Figure 1. Water Whirler 2005 (courtesy of the Len Lye Foundation)

## 2 NOTATION

$A$	Cross Sectional Area	$P$	Material Life Performance Index
$C$	Damping Matrix	$R$	Radius of Curvature
$C_D$	Drag Coefficient	$r_o$	Outer Radius of Cantilever
$d$	Element Path Length	$r_i$	Inner Radius of Cantilever
$E$	Elastic Modulus	$S$	Loss Factor
$E_k$	Kinetic Energy	$U_{aero}$	Cyclic Aerodynamic Damping Energy
$E_p$	Potential Energy	$U_{matl}$	Cyclic Material Damping Energy
$F(\omega)$	Frequency Domain Forcing	$x$	Cantilever Displacement Vector
$f$	Time Domain Forcing	$\dot{x}$	Cantilever Velocity Vector
$g$	Gravitational Acceleration	$t$	Time
$I$	Second Moment of Area	$X(\omega)$	Frequency Domain Displacement
$K$	Stiffness Matrix	$\sigma$	Stress
$K_b$	Cantilever Base Stiffness	$\rho$	Cantilever Density
$LR$	Loss Ratio	$\rho_{air}$	Air Density
$l$	Bending Length of Cantilever	$\gamma$	Stability Factor
$M$	Mass Matrix	$\gamma_{crit}$	Stability Factor at Critical Buckling Load
$m$	Mass of the Cantilever	$\omega$	Frequency

## 3 MATHEMATICS FOR MAINTAINING SIMILARITY WHILE INCREASING CANTILEVER SIZE

Lye produced sketches and written descriptions for both 'Water Whirler' and 'Flaming Harmonic'. Although he never built either of these sculptures, he did build a similar work called 'Harmonic'. The original 'Harmonic' is a vertical cantilever approximately 1.3m tall and made from 2.8mm diameter carbon steel rod. The base of the rod is held in a fixed clamp and reciprocates back and forth by a shake-table type drive mechanism created from an orbital sanding machine.

The purpose of this section is to establish the rules for changing the size of the sculpture. Observations of 'Harmonic' in the laboratory can be used to establish the design information required to build a larger version of 'Flaming Harmonic'. A 1.3m model of 'Harmonic', Figure 2a, was tested in a laboratory and the vibratory characteristics were discussed with the Len Lye Foundation. Based on these discussions, the design specifications for 'Flaming Harmonic' were established.

### 3.1 Dynamic Performance of the sculpture

Lye's drawings show that 'Water Whirler' produces two distinct vibratory forms during an artistic performance. Experimental testing showed that both of these forms could be created by oscillating a cantilever on a simple reciprocating shake table. The first form is a two-dimensional shape, Figure 2b, corresponding to the second beam bending mode (McCallion, 1972). Lye referred to this two-dimensional shape as the 'planar' mode. The second form, known as the 'whirling' mode, is a revolved version of the two-dimensional shape. This whirling form is created by slowly decreasing the shake table frequency from the planar mode by a few 10ths of a cycle-per-second, until the cantilever rod begins to whirl.

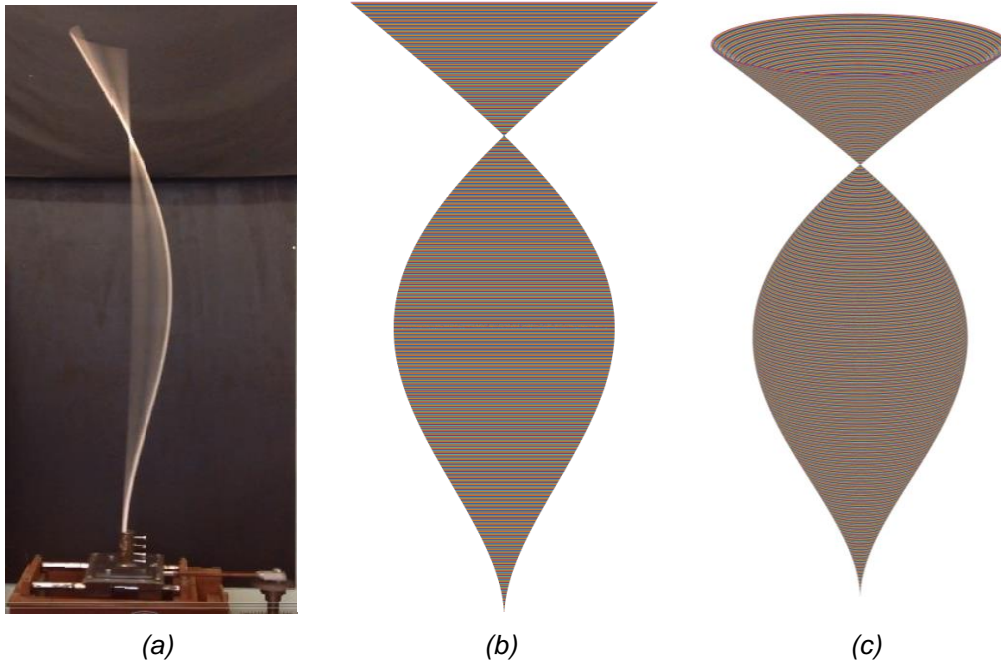


Figure 2. (a) 1.3m Fixed clamp 'Harmonic' on the shake table; (b) 2D planar mode; (c) 3D whirling mode

We describe the geometric shape of the whirling mode in terms of the planar mode by using the trigonometric shape function

$$\mathbf{x}_{3D} = \sin(2\pi\omega t) \mathbf{x}_{2D} + \cos(2\pi\omega t) \mathbf{x}_{2D} \quad (1)$$

The planar form is described as  $\mathbf{x}_{2D}$ , which is a vector of displacements corresponding to nodes up the length of the wand, found using a lumped-mass finite element method described in Section 4.

### 3.2 Maintaining Similarity

When establishing the design requirements, it became apparent that the mode shapes for the full-sized vibrating 'Flaming Harmonic' should be geometrically similar to those of the model-sized vibrating 'Harmonic'. Similarity conditions are used to ensure that different scale versions of a structure have the same set of properties; in other words, that a larger-scale wand will remain faithful to the smaller models that Lye produced.

### 3.3 Buckling Stability

Lye experimented with thin rods of spring steel in a local workshop near his residence in New York, USA (Horrocks, 2002). He tested the stability of various rods by holding them vertically and shaking them to form mode shapes. He would increase the length of the vertical cantilever rod until it buckled under its own weight, then he would lower the rod to obtain the correct stability for the performance he desired. Lye's intuitive experimental method can be replicated mathematically by quantifying the stability of a vertical cantilever at the design height and comparing this with the stability at the onset of buckling. Timoshenko and Gere (1961) define the dimensionless gravity parameter for a vertical cantilever as

$$\gamma = \frac{mgl^3}{EI} \quad (2)$$

According to Timoshenko and Gere, the critical value for the gravity parameter,  $\gamma_{cr}$ , at the point of buckling under self-weight, is 7.827. The approach to changing the size of the sculpture is to establish the gravity parameter of Lye's original sculpture and maintain the same ratio of  $\gamma/\gamma_{cr}$  between Lye's model and the sculpture built at a larger size.

Since the Flaming Harmonic uses a hollow tube to allow for a means for distributing the water and gas, the gravity parameter (2) may be written in the form

$$\gamma = 4\rho gl^3 (r_o^2 - r_i^2) / E(r_o^4 - r_i^4) \quad (3)$$

For the purpose of design there are two radii which require selection so that they satisfy Equations (2 and 3). Suitable values can be found by solving the quartic polynomial:

$$0 = \gamma E r_o^4 - 4\rho g l^3 r_o^2 + 4\rho g l^3 r_i^2 - \gamma E r_i^4 \quad (4)$$

By solving Equation (4) for  $r_o$  with a test value of  $r_i$ , selecting the largest positive, real root returns a practical value of  $r_o$  which satisfies the buckling stability criteria.

### 3.4 System Damping

Damping arises from two significant sources in the system, and it is useful to consider the Loss Ratio (LR) of energy in the system. The Loss Ratio may be defined as the energy lost per cycle due to aerodynamic and internal material losses divided by the sum of the kinetic and potential energy, namely

$$LR = \frac{U_{aero} + U_{matl}}{E_k + E_p} \quad (5)$$

If a pair of radii for the scaled cantilever are selected which satisfy Equation (2), then consideration of the loss ratio (Equation (5)) gives an understanding of the implications for adding energy to the system in order to achieve the same deflected shape for the vibrating cantilever. If the same loss ratio is achieved between the model and the full-size sculpture, then damping similarity is obtained.

The rotational motion of the cantilever in the whirling mode is at a constant rotational velocity,  $\dot{x}_i$  (where  $i$  represents an element in the vector  $\mathbf{x}$ ). Hence Reynolds number, kinetic energy and potential energy are constant with respect to time. For the planar mode, analysis of the damped energies,  $U_{aero}$  and  $U_{material}$ , is more complex because the speed of the cantilever changes during each cycle. The energy lost per cycle due to aerodynamic damping may be defined as the aerodynamic force on the cantilever, multiplied by distance travelled as it vibrates

$$U_{aero,i} = \frac{1}{2} C_{D,i} \rho_{air} A_i \dot{x}_i^2 d_i \quad (6)$$

The 'loss-factor' method, described by Carfagni et al. (1998), may be used to determine the material damping as

$$U_{material,i} = 2\pi S E_{k,max,i} \quad (7)$$

An estimate of the material loss coefficient,  $S$ , was found from graphical information produced by Ashby (2005). Carfagni et al. (1998) recognise that the method is only suitable near resonance, and other damping metrics are better for use when the cantilever is non-resonant. For Lye's 'Harmonic', where the cantilever is operated continuously at or near resonance, the loss ratio criteria is practically sufficient for damping similarity.

It is not always possible to hold the loss ratio between scales constant, as changing the material results in a different loss coefficient. Using a material with a large material loss coefficient may result in a loss ratio from material damping alone, which is higher than the loss ratio due to combined material and aerodynamic damping for the original 'Harmonic'. If this occurs, the loss ratio is still a useful a metric to consider in the selection radii pair.

## 4 THE VIBRATORY FORM

The boundary conditions for the base of the vertical cantilever were determined experimentally and in conjunction with the Len Lye Foundation. Two sets of boundary conditions are considered in this paper which each produce slightly different, but artistically acceptable planar and whirling mode shapes. These boundary conditions effect the fatigue life and practical limit to which the work can be produced (Spencer and Gooch, 2014). The first boundary condition considered is a fixed-free cantilever where the cantilever rod is in a fixed clamp in the vertical position to which a reciprocating sliding ground motion is applied, Figure 3b. The second boundary condition is a pinned-free cantilever where the cantilever rod is held in a pivot clamp, Figure 3c.

A finite-element approach was adapted from Newland (1989) to find the resonant frequencies and corresponding mode-shapes for the fixed and pivoting clamp designs. Figure 3a shows the basic model which includes four rigid links connected by springs and dampers at each joint. The equation of motion for the forced damped vibration of the system takes the general form:

$$m\ddot{x} + c\dot{x} + kx = f \quad (8)$$

Following the methodology in Newland (1989) the mass ( $M$ ) and stiffness ( $K$ ) matrices for the vertical cantilever are:

$$M = \frac{ml}{4} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 4 & 3 \\ 1 & 4 & 8 & 5 \\ 4 & 8 & 12 & 7 \end{bmatrix}, \text{ and } K = \frac{mg}{2} \begin{bmatrix} 0 & \frac{2k}{mgl} & \frac{-4k}{mgl} + 1 & \frac{2k}{mgl} - 1 \\ \frac{2k}{mgl} & \frac{-4k}{mgl} + 3 & \frac{2k}{mgl} - 2 & -1 \\ \frac{-4k}{mgl} + 5 & \frac{2k}{mgl} & -2 & -1 \\ \frac{-2K_b}{mgl} - 2 & -2 & -2 & -1 \end{bmatrix} \quad (9)$$

where  $k = \frac{EI}{l}$

A harmonic forcing function was applied to the clamped end to establish the response of the system. The harmonic forcing function is of the form

$$f = A \sin(2\pi\omega t) \quad (10)$$

The system of equations were solved by finding the periodic response of the dynamic system, by taking a Fourier transform on Equation (8) leading to the frequency response function (Gavin, 2016)

$$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{-\omega^2 M + \omega C + K} \quad (11)$$

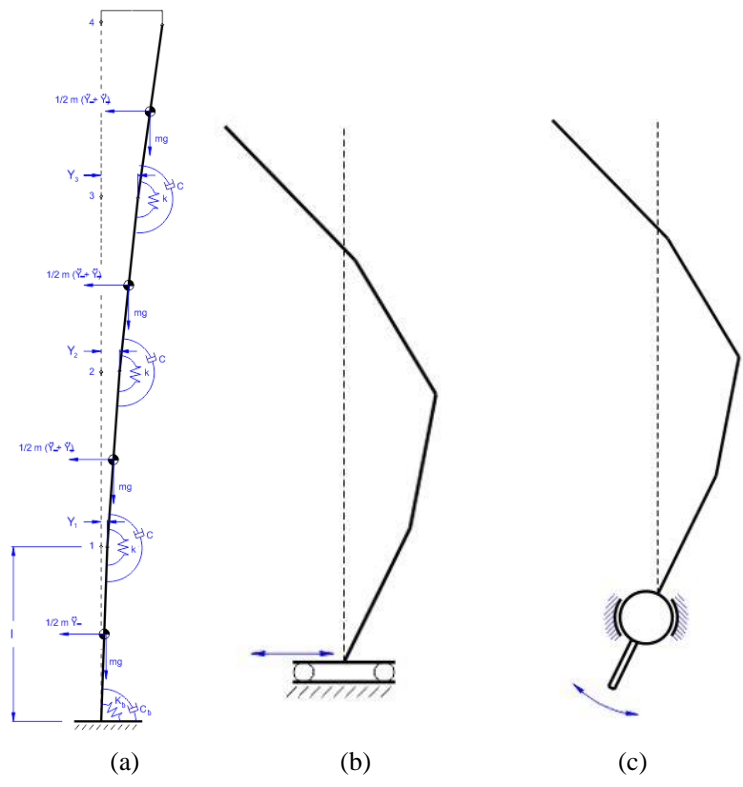


Figure 3. (a) Finite-element model (b) Fixed reciprocating clamp (c) Pivoting clamp

The time domain response of the structure was determined using the method outlined by Gavin (2016). The time domain response was solved using the MATLAB computing environment (version R2106A). The natural frequencies were found by plotting the frequency response function Equation (11) and noting the frequencies where the peak values occur.

Applying the end conditions to the lumped-mass model is achieved by modifying the stiffness matrix  $K$  (Equation (9)); e.g. a pinned end condition can be applied by setting the base stiffness,  $K_b$ , to zero.

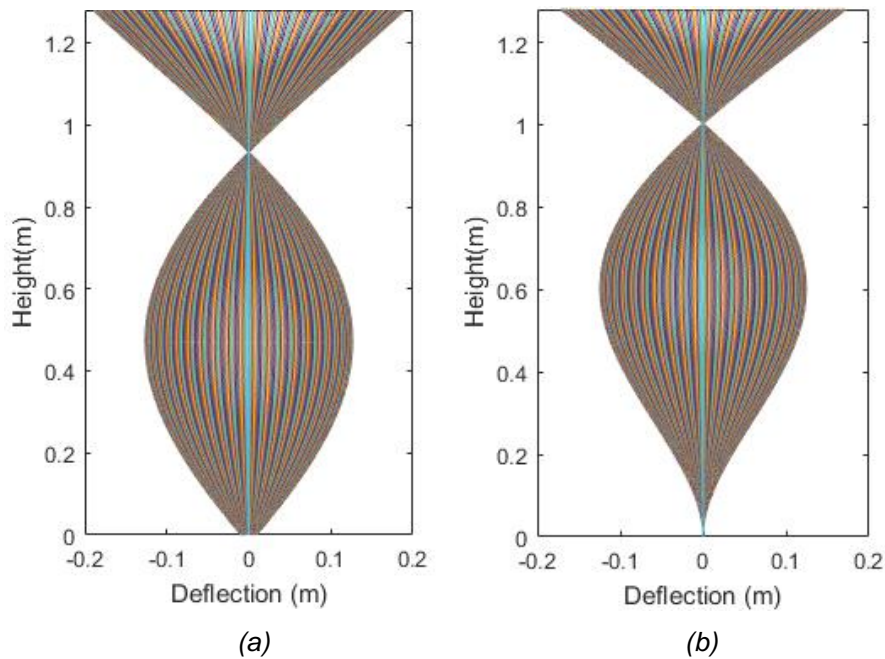


Figure 4. Comparison of the predicted mode shape and displacements for the forced-vibrations of (a) the pivot clamp cantilever and (b) the fixed clamp cantilever

#### 4.1 Softening Spring Effect

The predicted geometric shape of the fixed clamp cantilever was found to be consistent with both experimental observations and results found in the literature (McCallion, 1973). The resonant frequency of Lye's original Harmonic was measured to be 7.49 Hz using a Linear Variable Differential Transformer (LVDT) linear position sensor (sample rate of 100Hz). This is less than the frequency predicted by the lumped mass finite-element method (7.64 Hz). The finite-element method was verified using the gravity parameter method proposed by Naguleswaran (1991), which also predicted a frequency of 7.64Hz. The difference between the modelled and experimentally found frequency is explained by non-linear effects arising from the large amplitude of the vibration. The large amplitude causes a softening spring effect described by Duffing's Equation for non-linear oscillators (Newland, 1986).

Our experiments showed that it was possible to reach peak resonance by increasing the frequency slowly, as found by Malatkar (2003) for a third bending mode resonance. Practically, peak resonance is much more readily achieved by decreasing the frequency from above the resonance zone.

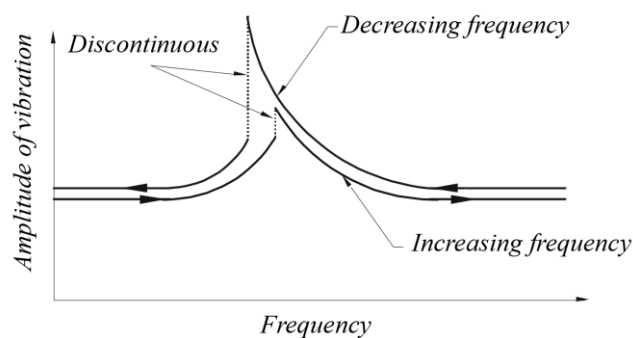


Figure 5. The softening spring effect

The description of the softening spring effect in Duffing's Equation implies that the planar and whirling modes of Lye's sculpture are the same resonance mode, at different amplitudes on the frequency response function (Figure 5). The cantilever resonates, and as the amplitude builds, the frequency can be dropped to increase the amplitude of the motion. Eventually, the system reaches a point where the resonance collapses.

## 4.2 Stress State of the Structure

The bending moment at any point along the length of the vertical cantilever can be determined from the radius of curvature using (Philpot, 2013) as

$$M = \frac{EI}{R} \quad (12)$$

Similarly, the bending stress is (Philpot, 2013)

$$\sigma = \frac{Er_o}{R} \quad (13)$$

By substituting the local curvature into Equations (12 and 13), the stress at any point along the cantilever can be found.

Figure 6 shows the curvature (defined as  $1/R$ ), which is directly proportional to the flexural stress distribution along the vertical cantilever (Equation (13)).

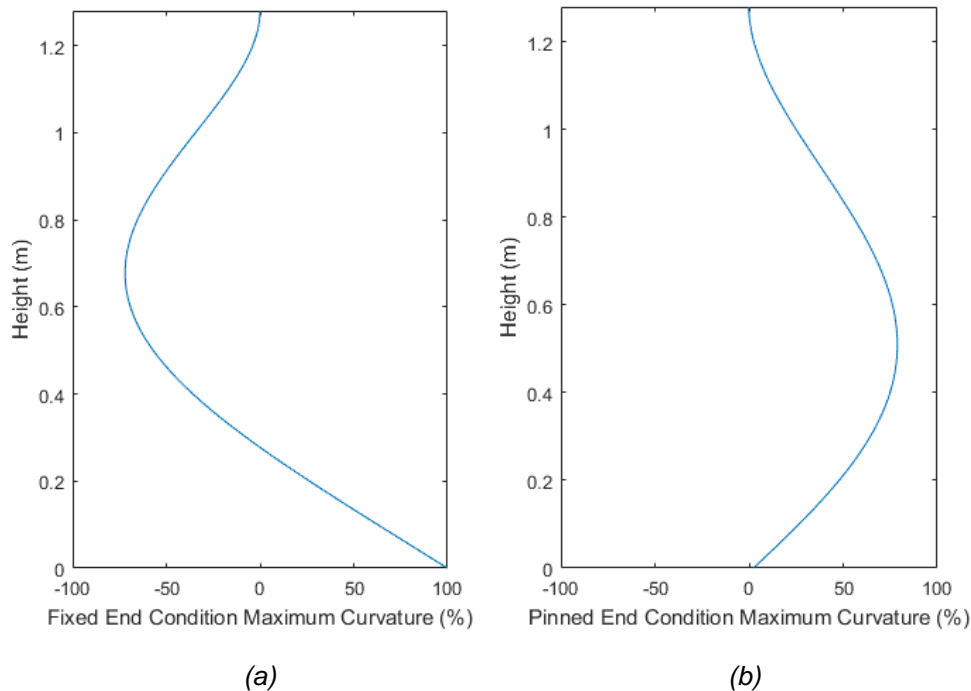


Figure 6. Comparison of curvature ( $1/R$ ) for the (a) fixed clamp and (b) pivot clamp

The stress maxima occur at the point of greatest curvature for each end condition. From Figure 6 it can be noted that this maximum stress occurs at the base for fixed clamp and 38% up from the base for the pivot clamp.

## 5 IMPLICATION OF SCALING FOR A STATICALLY SIMILAR VERTICAL CANTILEVER

This section establishes the implications of using the fixed (Figure 3b) or pivot (Figure 3c) clamp end conditions.

### 5.1 Validation

The original 'Harmonic' was made from 2.8mm diameter carbon steel rod (a section of this was tested using a Vickers micro-hardness test, and found to have a yield strength of approximately 1200MPa) and its structural properties produce a dimensionless gravity parameter  $\gamma = 1.56$ . The displaced shape of 'Harmonic' was captured using a high speed video. The minimum radius of curvature was estimated by superimposing an arc over the image of the vertical cantilever using the CAD program SolidWorks. From the measured radius of curvature, the bending moment and bending stress at the base of the fixed clamp vertical cantilever were estimated using Equations (12 and 13) above. A bending moment of 1.5Nm and normal bending stress of 703MPa were determined.



## 5.2 Material Selection

Following the approach set out in Ashby (2005), the gravity parameter Equation (2) can be used as a functional constraint for a performance equation based on minimising bending stress

$$P = \frac{1}{4} \left( \frac{\gamma}{g} \right) \left( \frac{R^2}{l^3} \right) \left( \frac{\sigma^2}{E\rho} \right) \quad (14)$$

The first two terms in Equation 14 are fixed by the desired geometric shape, here defined by the proportions of Lye's original vertical cantilever. However, the material properties index,  $\sigma^2/E\rho$ , is free and dependent on available materials. Searching for a material with the maximum value of the materials properties index is expected to result in the optimal life of the sculpture. As such, materials with both high strength-to-weight and strength-to-stiffness ratios are good candidates for the cantilever. Fibre reinforced plastics (FRPs) are by far the most promising options, with exotic alloys and some plastics being possible second-tier choices.

Fibreglass has been selected here as the material for construction of the vertical cantilever due to wide availability, low cost, and manufacturability. While the authors acknowledge extreme variability in the properties of all FRPs, Australasian manufacturers have been contacted, with reasonable values used in this analysis. Homogeneity of the materials along the length of the cantilever is an issue, and can only be quantified through mechanical testing for the specific manufacturing method used, on a case-by-case basis. Interesting future work exists in determining the impact of varying stiffness on the dynamics of the cantilever.

## 5.3 Structural and Vibrational Properties of a Scaled Cantilever for 'Flaming Harmonic'

The structural properties for a 10, 12 and 14m fibreglass cantilevers are summarised in Table 1. In each case, static similarity has been maintained between the original steel cantilever rod and the larger fibreglass cantilever rod using Equation 2, with the amplitude of the motion in the larger cantilevers scaled proportionally to the increase in length.

Table 1. Structural properties of scaled vertical cantilevers

L [m]	OD [mm]	ID [mm]	Mass [kg]	Pinned Stress [MPa]	Fixed Stress [MPa]
1.28	2.8	0	0.62	553.6	702.6
10	61.9	40	33.3	261	331.2
12	80.1	54.4	54.4	281.7	357.5
14	100	70	106.5	301.2	382.3

For the 10, 12 and 14m sculptures, the calculated Reynolds number is in the drag-crisis region (i.e.  $Re = 10^5 - 10^6$ ); as a result, non-linear variations in the drag coefficient are expected (White, 2011). The drag force coefficient for each beam element needs to be determined based on the whirling velocity of each element.

It is clear from Table 1 that using a pivot clamp design is advantageous due to the lower flexural stress in the cantilever.

## 6 CONCLUSION

Written descriptions of how the artist built the original sculpture, along with observations of the model-size 'Harmonic', have been sufficient to produce the design specification of the large scale sculpture.

Maintaining static similarity with the larger size 'Flaming Harmonic' is the best way to preserve the artistic integrity of the sculpture. The scaling rules presented in this paper are likely to replicate the artist's original intuitive experimental methods.

The pivot clamp design produces a vibrating mode shape close to the observed shape of the fixed clamp design in the original 'Harmonic'. The slight difference in shape between these two clamping methods has been deemed acceptable from an artistic viewpoint.

The pivot clamp design is likely to produce a reduced normal bending stress in the cantilever rod when compared with the fixed clamp design. However, a potential issue for the pivot clamp design may occur if the clamp and the vibrating rod become out of phase. In such a case this could result in a much higher

than expected bending stress at the clamp exit point. This issue may be alleviated by incorporating a slipping device such as a clutch in the mechanism for the drive design.

Careful design of the clamp will be necessary to minimise the stress concentration at the clamp exit point. This especially critical for the rod in the fixed clamp design, where the maximum bending stress is expected at the clamp exit point.

A non-linear softening spring effect has been observed in Lye's original 1.3m model. This effect necessitates a means for changing the excitation frequency to achieve the maximum amplitude of vibration.

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