DECENTRALIZED OPTIMAL DESIGN USING THE LAGRANGIAN RELAXATION APPROACH

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ABSTRACT
To minimize the coordination efforts among design teams and expedite the design process via parallel workflow, this paper proposes a framework of decentralized optimal design. The framework consists of two major components. The first component is the characterization of team-based interactions in engineering design via the notions of responsibility and controllability. This characterization helps us to understand what kind of role a team can perform in a decentralized design environment. The second component is the application of the Lagrangian relaxation approach to support concurrent decision making (i.e., parallel workflow) during the team-based design process. The framework has been demonstrated through the welded beam design example, and the results are promising in view of future development in this research direction.

Keywords: decentralized decision making, optimal design, Lagrangian relaxation

1 INTRODUCTION

1.1 Background
Due to the market's needs and pressure, modern engineered products tend to be more complex than ever. As such, multidisciplinary teams are required to handle different aspects of a product during the design process. In an ideal situation, these multidisciplinary teams should work seamlessly and cooperatively as a unified team to achieve the common good of a product. However, such kind of cooperation is often considered "too luxury" as it implies expensive organization and communication costs. The complexity of organization may also hinder the efficiency of the design process. In addition, these teams may have diverse domain backgrounds, and how to communicate their design decisions is always challenging. Therefore, the autonomy of a team is usually encouraged during the design process in practice. Then, an individual team can work on its own as much as it can, while their individual decisions can somehow collectively contribute to a better design overall.
In this work, the term "optimal design" is applied since optimization formalism is applied to formulate a design problem as an optimization problem [1]. In this context, a design process is viewed as the process to determine the values of n design variables (x_i), which can be expressed as a vector (i.e., $\boldsymbol{x}$ = [x_1, x_2, …, x_i, …, x_n]). A general design problem is formulated as follows.

General Optimal Design Formulation

$$\min_{\boldsymbol{x}} f(\boldsymbol{x})$$
$$\text{subject to } \boldsymbol{g}(\boldsymbol{x}) \le 0$$
$$\text{where } f : R^n \to R \text{ and } \boldsymbol{g}(\boldsymbol{x}) : R^n \to R^m \tag{1}$$

where f is the objective function and $\boldsymbol{g}$ is a vector of m constraints. Given the Formulation (1), decentralized optimal design is referred to the design situation, in which design variables can be determined via a number of decision makers. The term "decentralized" is applied to emphasize the autonomy of each decision maker during the design process. In other words, we want to investigate the process, in which a centralized coordinator is absent or does not perform any optimization duty. Decision makers need to make design decisions subject to limited information as well as limited interactions with other decision makers. This purpose of this research is to investigate a systematic approach to minimize the communication among teams while attempting the overall optimal design in practice.

1.2 Literature Review

Concerning the research efforts related to decentralized design problems, three different approaches have been identified: the hierarchical optimization approach, the game-theoretic approach, and the decomposition approach in optimization. The first two approaches mainly stem from the engineering design community, and the third approach from applied mathematics.

The hierarchical optimization approach invokes a hierarchical structure for solving large-scale optimization problems, in which the top level usually handles the design objective and the bottom level are responsible for satisfying constraints. One relevant research area is multidisciplinary design optimization (MDO), which utilizes a two-level computing structure, namely, a system level and a disciplinary level. Then, different MDO approaches have been proposed by specifying the roles of system and disciplinary levels in the optimization process [2]. Based on the MDO efforts, Braun [3] proposed the framework of collaborative optimization (CO), in which the disciplinary autonomy is emphasized to reduce the workload at the system level. Since then, various CO formulations have been proposed [4], [5]. However, Alexandrov & Lewis [6] found that CO might return undesirable results since convergence of CO has not been proven. In this context, the framework of target cascading [7] was proposed, which was supported by a convergence proof.

In the game-theoretic approach, a decentralized design problem is characterized as a multi-player game, in which each team is considered as a game player [8]. Then, the concepts from game theory are adapted to model and analyze team-based design [9]. Particularly, three game protocols are utilized (i.e., Pareto cooperation, Nash non-cooperation and Stackelberg leader/follower) to model and implement different coordination strategies in team design. Chen & Li [10] proposed the concepts of responsibility (scope of concern) and controllability (scope of control) to unify different game-theoretic models in design. Li et al. [11] have proposed three models to capture different cooperation modes in team-based design. Recent research on the game-theoretic approach can be found in [12], [13].

The research of the decomposition approach for large-scale optimization is based on some specific problem structures to expedite the optimization process [14]. A common problem structure is that an optimization problem contains a small set of so-called complicating variables or constraints. If these complicating variables or constraints are once removed, the original problem can be decomposed into a set of independent sub-problems, which can be solved independently [15]. The initial result in this research area can be found in the Dantzig-Wolfe decomposition principle [16], which addressed complicating constraints in linear programming. Then, the Benders algorithm [15], [17] was proposed to handle the complicating variables, and the Rosen algorithm [18] to handle both complicating constraints and variables. These research efforts were then generalized via the Lagrangian relaxation technique to address the traveling-salesman problem [19], integer programming [20] and nonlinear programming [21].

In contrast to the hierarchical optimization approach, this paper is intended to investigate the solution process that does not require a high-level team to perform optimization. Also, it is expected to propose a solution process that supports concurrent decision making or parallel workflow (i.e., teams can make design decisions at the same time without intensively communicating with other teams). This topic has not been thoroughly addressed in the game-theoretic approach. We resort to the decomposition approach for large-scale optimization, specifically, Lagrangian Relaxation (LR), to investigate the solution process for decentralized optimal design.

1.3 Paper's Motivation and Outline

The purpose of this paper is to propose a framework for decentralized optimal design, including formulation and execution. The formulation of decentralized optimal design will be provided in Section 2. Section 3 will discuss the Lagrangian relaxation approach for the execution of concurrent decision making among teams. Section 4 will illustrate the solution procedure via the welded beam design. Section 5 will provide closing remarks of this paper.

2 FORMULATION OF DECENTRALIZED OPTIMAL DESIGN

2.1 Problem Partitioning

Consider a general design problem formulated in (1). This problem is partitioned into t sub-problems, each of which is handled by an individual decision-making unit, namely, a team. It is assumed that each team is responsible for one objective, which depends on its own variables as well as variables of other teams. Using the notation given in Table 1, a partitioned problem is formulated as follows.

Formulation of a Partitioned Problem

$$\min_{\boldsymbol{x}} F\left(f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_t(\boldsymbol{x})\right)$$
$$\text{subject to } \boldsymbol{g}_c(\boldsymbol{x}) \le 0,\ \boldsymbol{g}_1(\boldsymbol{x}_1) \le 0,\ \boldsymbol{g}_2(\boldsymbol{x}_2) \le 0, \ldots,\ \boldsymbol{g}_t(\boldsymbol{x}_t) \le 0$$
$$\text{where } F(\cdot): R^t \to R,\ f_k(\boldsymbol{x}): R^n \to R,\ \boldsymbol{g}_c(\boldsymbol{x}): R^n \to R^{m_c},\ \boldsymbol{g}_k(\boldsymbol{x}_k): R^{n_k} \to R^{m_k} \quad (2)$$

Table 1. Notation for a partitioned problem

f_k	Local objective function of the *k*th team
F	Overall objective function that aggregates the objectives of all teams
$\boldsymbol{g}_c$	Vector of common constraint functions
$\boldsymbol{g}_k$	Vector of local constraint functions of the *k*th team
m_c	Number of common constraints
m_k	Number of local constraints of the *k*th team
n_k	Number of variables of the *k*th team
$\boldsymbol{x}$	Vector of variables of all teams
$\boldsymbol{x}_k$	Vector of variables determined by the *k*th team

For simplicity, we denote ($\boldsymbol{x}_k$, $\boldsymbol{x}_{-k}$) as the vector in which variables pertaining to $\boldsymbol{x}_k$ can be controlled by the *k*th team, while other variables ($\boldsymbol{x}_{-k}$) are kept as constants. Then, the formulation of the sub-problem that is handled by the *k*th team is given below.

Sub-problem Formulation (Individual Objective)

$$\min_{\boldsymbol{x}_k} f_k(\boldsymbol{x}_k, \boldsymbol{x}_{-k})$$
$$\text{subject to } \boldsymbol{g}_c(\boldsymbol{x}_k, \boldsymbol{x}_{-k}) \le 0,\ \boldsymbol{g}_k(\boldsymbol{x}_k) \le 0 \quad (3)$$

Assuming that all teams take their efforts to minimize the aggregated objective, *F*, a sub-problem is formulated below.

Sub-problem Formulation (Aggregated Objective)

$$\min_{\boldsymbol{x}_k} F\left(f_1(\boldsymbol{x}_k, \boldsymbol{x}_{-k}), f_2(\boldsymbol{x}_k, \boldsymbol{x}_{-k}), \ldots, f_t(\boldsymbol{x}_k, \boldsymbol{x}_{-k})\right)$$
$$\text{subject to } \boldsymbol{g}_c(\boldsymbol{x}_k, \boldsymbol{x}_{-k}) \le 0,\ \boldsymbol{g}_k(\boldsymbol{x}_k) \le 0 \quad (4)$$

Given the sub-problem formulation in (4), collective team efforts are expected to determine the values of all variables to optimize the common objective and satisfy both common and local constraints.

2.2 Characterization of Team-based Interactions

Given the partitioned problem formulated in (2), the next question is how to solve the sub-problems (as formulated in (3) and (4)) in such a way that each decision maker can work on their sub-problems autonomously without intensive interactions with other teams. In this context, this section is intended to characterize the typical modes of team-based interactions. These characterized team-based design interactions will be used to implement the solution process for decentralized optimal design.

To describe different modes of interactions, each team is characterized via two notions: responsibility and controllability [10]. A team's responsibility is referred to as the scope of design objective(s) that the team needs to consider or pursue. For instance, suppose that a team design process consists of two teams: a design team whose objective is about a product's functions and a manufacturing team whose objective is about the manufacturing cost. If a design team is required to consider the manufacturing cost during its own decision-making process, the design team is considered as having a high (or global) responsibility since it needs to consider both the design and manufacturing objectives. In contrast, if the manufacturing team only concerns about the manufacturing cost, it is considered as having a low (or local) responsibility.

A team's controllability is referred to as the range of design variables the team is entitled to determine or control. Originally, each team has its own design variables, and the team's variables are disjoint from one another. However, for instance, if a design team, in addition to its own design variables, directly influences or controls the decision about which manufacturing processes to be used (this decision is also considered by the manufacturing team), the design team is considered as having a high (or global) controllability. In contrast, the design team just focuses on its own design variables exclusively, it is considered as having a low (or local) controllability.

Having the notions of responsibility and controllability, we can characterize four different modes of team interactions for decentralized optimal design. Notably, though our prior work has derived more modes of team interactions [10], [11], we only investigate the modes in this paper that can be meaningfully interpreted in the context of decentralized optimal design. Suppose that we have two teams in the design process. The four modes of interactions are listed in Table 2. The first mode represents seamless cooperation, in which all teams are concerned with the same aggregated objective (i.e., high responsibility) with global control of all design variables (i.e., high controllability). This situation is similar to having a unified team to resolve a single optimization problem (i.e., cooperation and intensive communication are allowed in the design process). This ideal situation is not intended for decentralized optimal design but such situation can be used as a benchmark to evaluate different schemes for decentralized optimal design.

Table 2. Different modes of team interactions

Team 1		Team 2		Mode of interactions
Responsibility	Controllability	Responsibility	Controllability	
High	High	High	High	Seamless cooperation
High	Low	High	Low	Collaborative interaction
Low	Low	Low	Low	Non-cooperative interaction
High	Low	Low	Low	Overseeing leader interaction (Team 1)

The other three modes of interactions can be viewed as different situations in decentralized optimal design. One common feature of these modes of interactions is that they all invoke low controllability to describe a team's characteristics. In this context, low controllability can be considered as the characterization of a team's autonomy in a sense that teams only manipulate their own design variables without directly determining variables of other teams, thus reflecting the nature of decentralized optimal design.

The second mode of interactions in Table 2 is termed collaborative interaction, in which all teams are separated to locally control their own design variables to optimize the common aggregated objective (i.e., high responsibility). This situation captures that all teams are able to identify a clear, single objective, which can be optimized to benefit all teams. However, seamless communication is not allowed so that they can only control their own variables to achieve the same objective.

The third mode is termed non-cooperative interaction. In addition to the local control of variables, each team only concerns its own design objective. Since all teams cannot come up with a common objective in this case, they tend to determine design variables for their own interest. This situation is not unrealistic in practice. For instance, if the design problem is large and complex, it will not be obvious to articulate a common objective that all teams can understand and work with. Then, it is viable to have each team pursuing its own interest to achieve a good design overall. The premise is that no team wants to do something bad for the design intentionally.

The fourth mode belongs to the leader / follower interaction, in which the leader team has a higher responsibility (i.e., the overseeing leader). The overseeing leader (which is Team 1 according to Table 2) is limited to its own control of design variables to achieve the aggregated design objective. In the meantime, the follower team can be viewed as an "innocent" team, which only controls its own design variables to achieve its own objective.

2.3 Expectation from Different Modes of Interactions

As mentioned before, seamless cooperation should yield the best overall result compared with other modes of interactions but the drawback is the price of intensive communication. In contrast, sub-optimal results should be expected from the non-cooperative mode of interactions. In the collaborative mode of interactions, since all teams are willing to cooperate for the common objective, it should be easier to lead to a converged solution (compared to the non-cooperative mode). The major limitation in achieving optimal results is due to the lack of seamless communication.

To analyze the leader / follower case, suppose that the non-cooperative mode is employed, and a team's sub-problem (say, Team 1 according to Table 2) is solved according to Formulation (3). The resulting objective value of Team 1 is f_1^{NC}. Then, suppose that the same optimization model is applied, and the overseeing leader model is invoked in this case. It is expected that the resulting objective value (say, f_1^{OL}) will be not higher the objective value from the non-cooperative mode (i.e., $f_1^{OL} \leq f_1^{NC}$) due to higher responsibility.

Different modes of interactions have been studied in our prior research [10], [11]. Due to the existing of the common constraints (i.e.. $\boldsymbol{g_c}$ in Formulation (2)), our prior execution models only allow teams to make design decisions in an alternative manner (i.e., one team after another) to ensure the feasibility of the final solution. In this paper, the Lagrangian relaxation approach is applied so that concurrent decision making among teams is made possible.

3 LAGRANGIAN RELAXATION APPROACH

The basic notion of Lagrangian relaxation (LR) is to relax the original problem by removing the complicating constraints (or variables via the duality of the original problem [15]) and treating them as penalty terms (via Lagrangian multipliers) in the objective function. Then, a two-level computing structure is invoked: master level and sub-problem level. The master level is responsible for regulating the values of Lagrangian multipliers for satisfying the complicating constraints. In turn, each sub-problem is formulated with a set of local constraints and an objective function that includes Lagrangian penalty terms. In this way, the computational burden at the master level can be minimized since it does not need to deal with actual decision variables (which are handled at the sub-problem level) or run optimization to set target values for sub-problems. In other words, the master level in the LR approach only performs the *regulating* duty, rather than the *optimization* duty as the case in MDO and CO discussed in Section 1.2.

The intuition of the Lagrangian multipliers is similar to the penalty concept in optimization. Each Lagrangian multiplier can be viewed as a price associated with a common constraint. When the price's value increases, the team will tend to select the solution point that is far from the boundary of the associated constraint. Similarly, when the price's value decreases, the team will attempt the solution point closer to the constraint boundary. In this way, the price values can be regulated to coordinate the solution points of different teams to yield an overall optimal and feasible solution.

3.1 Formulation

Assume that there are t teams to minimize a common objective, f. Then, the global view of the optimization problem can be expressed as follows.

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_t)$$
$$\text{subject to } \boldsymbol{g}_c(\boldsymbol{x}) \leq 0,\ \boldsymbol{g}_1(\boldsymbol{x}_1) \leq 0,\ \boldsymbol{g}_2(\boldsymbol{x}_2) \leq 0, \ldots,\ \boldsymbol{g}_t(\boldsymbol{x}_t) \leq 0 \tag{5}$$

The above optimization problem can be solved by individual teams if the common (or complicating) constraints ($\boldsymbol{g}_c$) are removed (or relaxed). A Lagrangian function (symbolized as L) is then employed for relaxation purpose, and its formulation is given below [15].

$$L(\boldsymbol{x}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) + \boldsymbol{\lambda}^T \cdot \boldsymbol{g}_c(\boldsymbol{x}) \tag{6}$$

where $\boldsymbol{\lambda}$ is the Lagrangian multiplier vector. Given a feasible solution and $\boldsymbol{\lambda} \geq 0$, the term $\boldsymbol{\lambda}^T \cdot \boldsymbol{g}_c(\boldsymbol{x})$ is non-positive. In other words, if $f(\boldsymbol{x}) < L(\boldsymbol{x}, \boldsymbol{\lambda})$, the solution must be infeasible.

Suppose that the values of the Lagrangian multipliers are given (symbolized as $\bar{\boldsymbol{\lambda}}$). The optimization sub-problem of the kth team can be formulated as follows.

$$\min_{\boldsymbol{x}_k} f(\boldsymbol{x}_k, \boldsymbol{x}_{-k}) + \bar{\boldsymbol{\lambda}}^T \cdot \boldsymbol{g}_c(\boldsymbol{x}_k, \boldsymbol{x}_{-k})$$
$$\text{subject to } \boldsymbol{g}_k(\boldsymbol{x}_k) \leq 0 \tag{7}$$

Although the LR approach has been applied for decentralized optimization problems, it may experience difficulty in convergence [21]. In this context, we have proposed an objective adjustment factor (symbolized as μ_k) in the formulation [22]. As a result, the Lagrangian function of the *k*th team (symbolized as L_k) can be re-formulated as follows.

$$L_k = \mu_k \cdot f(\boldsymbol{x}_k, \boldsymbol{x}_{-k}) + \overline{\boldsymbol{\lambda}}^T \cdot \boldsymbol{g}_c(\boldsymbol{x}_k, \boldsymbol{x}_{-k}) \tag{8}$$

The intuition of the objective adjustment factor is as follows. In the LR approach, although all the teams are responsible for optimizing the original objective, they essentially search for a solution point based on the Lagrangian function in Formulation (6). If the Lagrangian terms (i.e., $\boldsymbol{\lambda}^T \cdot \boldsymbol{g}_c$) are substantially larger than the objective term (i.e., *f*), teams may not be able to directly get a solution point that yields a better objective value. Then, the objective adjustment factor is used to balance the effects from both the objective term (i.e., *f*) and the Lagrangian terms (i.e., $\boldsymbol{\lambda}^T \cdot \boldsymbol{g}_c$). In our previous research, we use the following formulations to quantify the objective adjustment factor.

$$r_i = \frac{\partial L / \partial x_i}{\partial f / \partial x_i} \qquad \mu_k \approx (r_{k1} \cdot r_{k2} \cdot \ldots \cdot r_{ka})^{1/a} \tag{9}$$

where r_i is the ratio of the Lagrangian sensitivity to the objective sensitivity based on the variable x_i. The value of the Lagrangian sensitivity is used to reflect how effective of the variable x_i to modify the Lagrangian function. On the other hand, the value of the objective sensitivity is used to reflect how effective of the variable x_i to modify the objective function. Then, this ratio r_i provides the essential means to adjust the strength of the objective term (*f*) in the Lagrangian function.

3.2 Solution Procedure

By referencing [15], the procedure of solving a decentralized team-based optimization problem is shown in Figure 1. In Step 1 of the procedure, the original design problem is formulated according to Formulation (1) and partitioned according to Formulation (2). In Step 2, the LR approach is invoked, and each team is required to solve a sub-problem according to the following formulation.

$$\begin{aligned} &\min_{\boldsymbol{x}_k} \mu_k \cdot f_k(\boldsymbol{x}_k, \boldsymbol{x}_{-k}) + \overline{\boldsymbol{\lambda}}^T \cdot \boldsymbol{g}_c(\boldsymbol{x}_k, \boldsymbol{x}_{-k}) \\ &\text{subject to } \boldsymbol{g}_k(\boldsymbol{x}_k) \le 0 \end{aligned} \tag{10}$$

Then, we initialize design variables ($\boldsymbol{x}_k$), Lagrangian multipliers ($\boldsymbol{\lambda}$), and the objective adjustment factor (μ_k) for each team in Step 3. In Step 4, each team solves their design sub-problem based on the given information. Then, we check whether all team's results converge to a single solution or not. If convergence has not been met yet, we update the Lagrangian multipliers according to the least design solution and solve the sub-problems at the team level again in Step 4. If convergence is met, we output the results.

To update the Lagrangian multipliers, different schemes have been proposed, such as the cutting plane method [23] and the bundle method [24]. In this paper, we employ the sub-gradient method [15], [25] since most of other approaches require the master level to perform optimization, thus defeating the original purpose of decentralizing. To update the Lagrangian multipliers at the *i*th iteration, the following formulation is used.

$$\boldsymbol{\lambda}^{(i)} = \boldsymbol{\lambda}^{(i-1)} + k^{(i)} \frac{\boldsymbol{g}_c(\boldsymbol{x}^{(i)})}{\left\| \boldsymbol{g}_c(\boldsymbol{x}^{(i)}) \right\|} \tag{11}$$

where $\boldsymbol{\lambda}^{(i)}$ is the Lagrangian multipliers at the *i*th iteration; $\|\boldsymbol{g}_c\|$ is the norm (or magnitude) of the vector $\boldsymbol{g}_c$; $k^{(i)}$ is a step size (a preset constant) at the *i*th iteration, which can be determined by the following formulation.

$$k^{(i)} = \frac{1}{a + b \cdot i} \tag{12}$$

where a and b are scalar constants, and they are set as a=1 and b=0.1 in this paper. It has been proven that if the step size follows the conditions formulated in (13) below (along with the convexity condition), the procedure based on the sub-gradient method will converge to the optimal solution in finite steps [21]. However, it has been stated that the convergence rate can be very slow [15].

$$\lim_{i\to\infty} k^{(i)} \to 0 \qquad \sum_{i=1}^{\infty} k^{(i)} \to \infty \qquad (13)$$

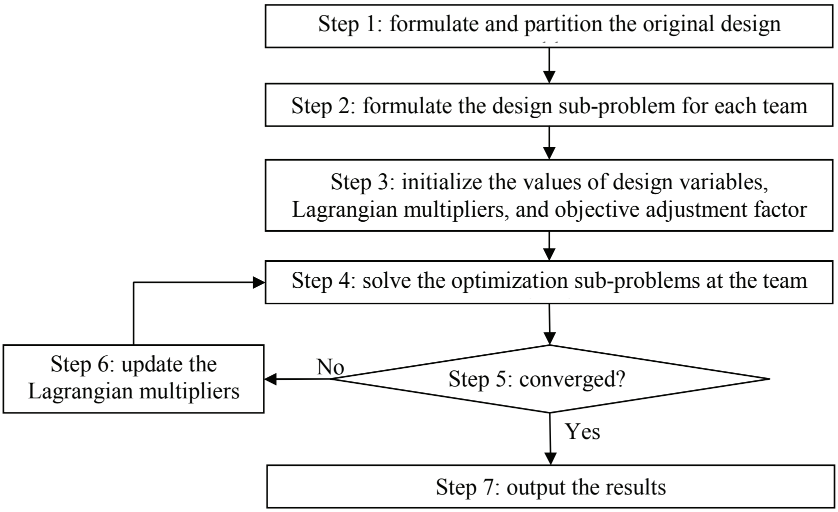

Figure 1.Solution procedure for solving decentralized design problems

3.3 Implication to Decentralized Optimal Design

While the LR approach can be numerically executed as described above, it also implicates how a team-based design process can proceed in a decentralized environment. Initially, it is supposed that design teams are set up, and each of them has their expertise and partial control on the final design, as well as a formulated objective that is suitable for the context of the team (e.g., high or low responsibility). Upon the consensus of the teams (or the company's policy), they periodically share their design decisions with each other so that they can update the current design progress at some specific points of the design process.

In the meantime, there exist some common constraints that are collectively affected by a number of teams. If these common constraints are violated, the involved teams will be *penalized* such that they will seek for the *conservative* solutions that will reduce the chance of violating the common constraints. Teams can receive different degrees of penalty for regulating the team's behavior towards the common constraints. Then, this penalizing mechanism can indirectly coordinate different teams to satisfy the common constraints.

In the algorithmic procedure, the penalizing mechanism is done through the Lagrangian terms in the objective function (i.e., $\boldsymbol{\lambda}^T \cdot \boldsymbol{g}_c$). By analogy, similar penalizing mechanism can be arranged according to the actual team design context (such as setting up the prices of violating common constraints and the corresponding price-update mechanism). In such a way, each team can still maintain its autonomy to complete its partial design, while it is *motivated* to avoid violation of common constraints.

4 WELDED BEAM DESIGN EXAMPLE

The welded beam design example is adapted from [26], and it is illustrated in Figure 2. In this example, two teams are set up: a beam team that determines the beam's geometry (i.e., t and b) and a weld team that determines the weld's geometry (i.e., h and l). Five design constraints are considered, and they are the beam deflection (δ), the bending stress in the beam (σ), the buckling load (F), weld geometry compatibility, and the shear stress in the weld (τ). The parametric constants of this welded beam design are listed in Table 3.

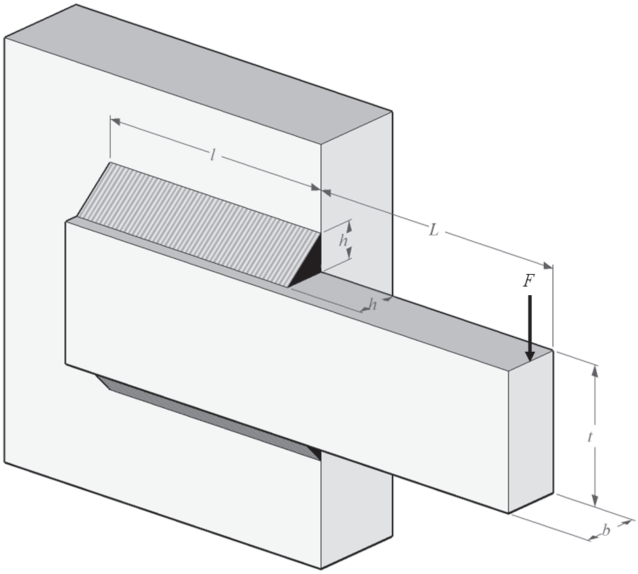

Figure 2.Illustration of the welded beam design

Table 3.Constants of the welded beam design

Parameter	Symbol	Value
Welding cost	c_1 (\$/mm^3)	6.741e-5
Beam material cost	c_2 (\$/mm^3)	2.936e-6
Beam's length	L (mm)	355
Applied force	F_o (kN)	27
Young's modulus	E (GPa)	206.84
Shear modulus	G (GPa)	82.737
Max. deflection	δ_{max} (mm)	6
Max. beam bending stress	σ_{max} (MPa)	200
Max. weld shear stress	τ_{max} (MPa)	90

After problem formulation and partitioning, the objective of the beam team is to minimize the beam material cost (f_1). Also, it is responsible for three local design constraints, which are related to the beam deflection (g_1), the bending stress in the beam (g_2), and the buckling load (g_3). The objective and constraints of the beam team are formulated as follows.

$$f_1 = c_2 tb\left(L + l\right) \tag{14}$$

$$g_1 = \frac{4F_0 L^3}{Et^3 b} - \delta_{\max} \le 0 \tag{15}$$

$$g_2 = \frac{6F_0 L}{bt^2} - \sigma_{\max} \le 0 \tag{16}$$

$$g_3 = F_0 - \frac{4.013\sqrt{EG(t^2 b^6 / 36)}}{L^2}\left(1 - \frac{t}{2L}\sqrt{\frac{E}{4G}}\right) \le 0 \tag{17}$$

On the other hand, the objective of the weld team is to minimize the welding cost (f_2) subject to two constraints which are common for both teams, i.e., weld geometry compatibility (g_4) and the weld shear stress (g_5). The objective of the weld team and the common constraints are formulated below.

$$f_2 = c_1 h^2 l \tag{18}$$

$$g_4 = h - b \le 0 \tag{19}$$

$$g_5 = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{l}{2R} + (\tau'')^2} - \tau_{\max} \le 0 \tag{20}$$

where $\tau' = \frac{F_0}{\sqrt{2}hl}$, $\tau'' = F_0\left(L + \frac{l}{2}\right)\left(\frac{6}{\sqrt{2}hl^3 + 3\sqrt{2}hl(h+t)^2}\right)\sqrt{\frac{l^2}{4} + \left(\frac{h+t}{2}\right)^2}$

Given the above formulations, several parameters are further set in order to execute the solution procedure, and they are listed as follows:

- Initial solution: $\boldsymbol{x}^{(0)} = [t^{(0)}, b^{(0)}, h^{(0)}, l^{(0)}] = [220,10,8,150]$
- Initial Lagrangian multipliers: $[\lambda_1^{(0)}, \lambda_2^{(0)}] = [1,1]$
- Variable bounds: $0.6\boldsymbol{x}^{(0)} \le [t, b, h, l] \le 1.4\boldsymbol{x}^{(0)}$
- $[\mu_1, \mu_2] = [200,600]$

Beam Team	Weld Team
Collaborative Model	
$\min_{t,b} \mu_1 f_1 + \mu_2 f_2 + \lambda_1 g_4 + \lambda_2 g_5$	$\min_{h,l} \mu_1 f_1 + \mu_2 f_2 + \lambda_1 g_4 + \lambda_2 g_5$
Subject to $g_1 \le 0$, $g_2 \le 0$, $g_3 \le 0$	Subject to $4.8 \le h \le 11.2$, $90 \le l \le 210$
$132 \le t \le 308$, $6 \le b \le 14$	
Non-cooperative Model	
$\min_{t,b} \mu_1 f_1 + \lambda_1 g_4 + \lambda_2 g_5$	$\min_{h,l} \mu_2 f_2 + \lambda_1 g_4 + \lambda_2 g_5$
Subject to $g_1 \le 0$, $g_2 \le 0$, $g_3 \le 0$	Subject to $4.8 \le h \le 11.2$, $90 \le l \le 210$
$132 \le t \le 308$, $6 \le b \le 14$	
Beam Leader Model	
$\min_{t,b} \mu_1 f_1 + \mu_2 f_2 + \lambda_1 g_4 + \lambda_2 g_5$	$\min_{h,l} \mu_2 f_2 + \lambda_1 g_4 + \lambda_2 g_5$
Subject to $g_1 \le 0$, $g_2 \le 0$, $g_3 \le 0$	Subject to $4.8 \le h \le 11.2$, $90 \le l \le 210$
$132 \le t \le 308$, $6 \le b \le 14$	
Weld Leader Model	
$\min_{t,b} \mu_1 f_1 + \lambda_1 g_4 + \lambda_2 g_5$	$\min_{h,l} \mu_1 f_1 + \mu_2 f_2 + \lambda_1 g_4 + \lambda_2 g_5$
Subject to $g_1 \le 0$, $g_2 \le 0$, $g_3 \le 0$	Subject to $4.8 \le h \le 11.2$, $90 \le l \le 210$
$132 \le t \le 308$, $6 \le b \le 14$	

Figure 3. Four models of decentralized optimal design for the welded beam design

Then, four models of decentralized optimal design are set, and the formulations are shown in Figure 3. Note that the main difference of these models lies in the formulations of the team's objectives. These models are run for 100 iterations according to the solution procedure in Figure 1. Figure 4 shows the plots of the overall cost (i.e., $f_1 + f_2$) at each iteration. The solution results after 100 iterations are listed in Table 4, which also shows the solution of seamless cooperation for comparison.

Figure 4 shows that all models exhibit an oscillation nature in the solution process. Since two teams are arranged to make design decisions in a decentralized manner, they need more iterations to exchange their design solutions before converging to a stable solution. Though all models show a converging behavior during the solution process, it is clear that the collaborative model has a narrower range of oscillation compared with the non-cooperative model. As mentioned in Section 2.3, since the collaborative model formulates the teams to share the common objective, it will be easier for these teams to converge to a stable solution.

Table 4.Numerical results of the welded beam design

Model	t (mm)	b (mm)	h (mm)	l (mm)	f_1 (\$)	f_2 (\$)	f_1+f_2 (\$)
Collaborative	215.49	6.19	5.71	90.00	1.7427	0.1978	1.9405
Non-cooperative	215.49	6.19	4.80	112.49	1.8308	0.1747	2.0055
Beam leader	215.49	6.19	4.80	112.49	1.8308	0.1747	2.0055
Weld leader	215.49	6.19	5.71	90.00	1.7427	0.1978	1.9405
Seamless cooperation	215.49	6.19	5.67	90.00	1.7427	0.1950	1.9377

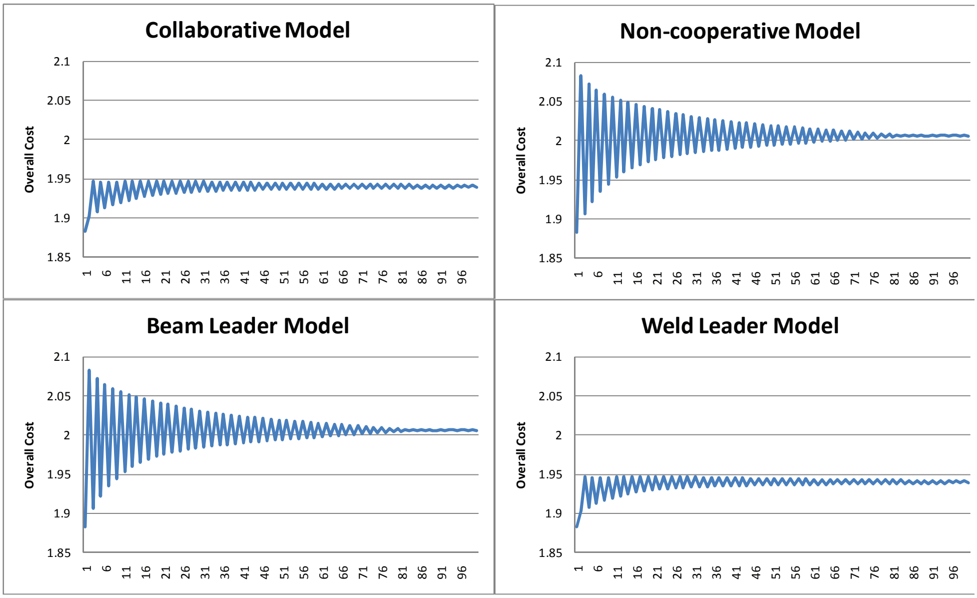

Figure 4.Plots of four interaction models in the welded beam design

In view of the optimality of the solution, Table 4 shows that both the collaborative and weld leader models yield the solutions that are close to the optimal solution (which is also shown in the last row of Table 4 in view of minimizing f_1+f_2 for comparison). It supports that the team-based design models are capable to approach to the optimal design in spite of the limited communication during the design process. Comparatively, the non-cooperative and beam leader models converge to a sub-optimal solution, which suggests the weld geometry with a smaller cross section and a longer length.

Notably, both collaborative and weld leader models yield the same intermediate solutions throughout iterations (not just the final solution), as well as the non-cooperative and beam leader models yield the same results. This observation can be explained by the fact that the weld team's objective (i.e., f_2 in Equation (18)) does not contain the variables of the beam team (i.e., t and b). Therefore, the term "$\mu_2 f_2$" in the beam's objective function can be simply viewed as a constant term during the optimization of the beam's sub-problem. As a result, in view of optimization, the collaborative model and the weld leader model are essentially equivalent, as well as the non-cooperative model and the beam leader model.

In addition, we have re-executed the non-cooperative model, and the model is able to yield solutions converging to the range of the optimal solution after about 160 iterations. The results are plotted in Figure 5. Even though it is not shown in the plot, the actual values of Lagrangian multipliers are in fact changing slightly after each iteration. Thus, after a certain number of iterations, the values of Lagrangian multipliers can alter the final solution from the sub-optimal range to the range of the optimal solution, as indicated in Figure 5. This observation implies that the non-cooperative model, in spite of its disadvantaged setup, can still lead to an optimal solution if sufficient numbers of iterations are allowed. Also, it implies that setting the stopping criteria can be challenging as the improper criteria can lead to pre-mature or sub-optimal solutions.

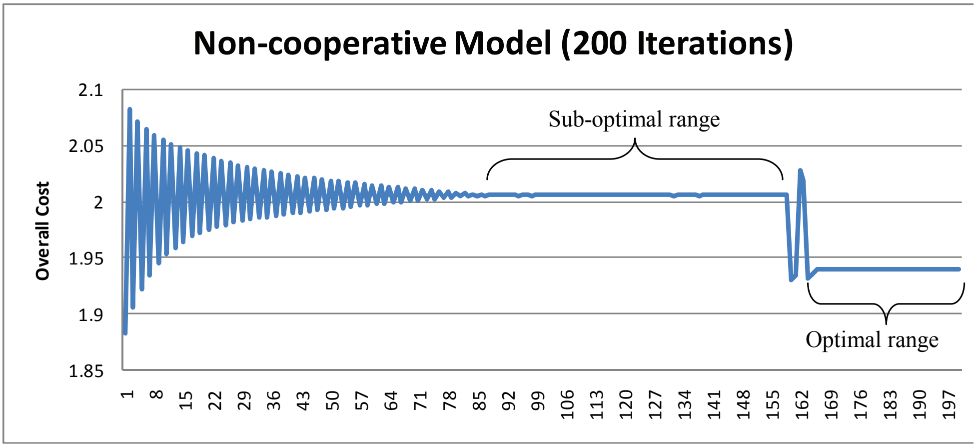

Figure 5.Plot of the non-cooperative model for 200 iterations

5 CLOSING REMARKS

In team design, common constraints are often the bottleneck for decentralized decision making. Particularly, a team may need to wait for another team's decision to avoid constraint violation. In this context, this paper contributes to investigating different characteristics of teams in interactions (via responsibility and controllability) and the possible scheme for concurrent decision making (via the Lagrangian relaxation (LR) approach). The intuition of the LR approach is to apply penalty terms to regulate teams' decisions to avoid violation of common constraints. The welded beam design example illustrates the feasibility of this concept.

The long-term goal of this research is to provide a systematic approach to construct an appropriate computing structure for coordinating teams and their design sub-problems to achieve the overall design. This paper has focused on parallel processes. However, both parallel and sequential processes should be coordinated for effective team design. The next step of this research is to investigate, given a large-scale complex design problem, how to practically propose an appropriate team-based computing structure that will lead to robust design results with minimum communication expenses. One direction of future work is to focus on utilizing the abundant results of the LR approach from applied mathematics to develop a robust framework for solving decentralized design problems.

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